Significance of Moment of Discrete Signals in Homomorphic Processing

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Moments are known as a set of descriptive constants for both continuous and discrete signals. For the convolution of two discrete signals, it is shown using the frequency domain definition of moment that the moments of cepstrum of the convolution output is the sum of moments of cepstrum of each signal component. Therefore, it is possible to deconvolve the signals in terms of moments. In the conventional homomorphic processing, one is usually concerned with the complex logarithm of the Fourier transform and must ensure its proper handling by using sophisticated algorithms such as phase unwrapping. Further, it is also required to calculate the infinite duration cepstrum using discrete Fourier transform techniques and therefore leads to unavoidable aliasing error. It is shown in this paper that the moments of a signal sequence and of its corresponding cepstral sequence are related in a way which obviates the need for the direct calculation of the cepstral coefficients. Hence an ideal calculation for the moment of cepstrum is possible even if the duration of the cepstrum is infinite. It is also possible to describe all the properties of a signal sequence from its moments. Effectiveness of this new method which is based on calculating the moments of a signal sequence and its cepstrum, is demonstrated in the problems of homomorphic deconvolution and echo removal. In linear filters, moments of minimum phase signal are related to moments of the corresponding linear phase signal using cepstral moments.

Key words: Cepstrum/Moment/Deconvolution/Aliasing error/Fourier transform

I. INTRODUCTION

In Signal Processing, moments are known as one set of descriptive constants which describe the given signal in its totality. In the image analysis, moment invariants are used in pattern recognition and several useful relations also exist which employ the invariant nature of moment under picture transformation [1]-[3]. However, in one dimensional discrete signal processing, effective application of moment is not much, the main reason being the laborious mathematics which takes away the practical aspect of moment and their effective use. Studies involving the moment sensitivities of the impulse response of the FIR filter, finite word length effects have also been done [4]. In our earlier work too [5],[6], we have described some of the concrete applications of moments concerning the discrete, one dimensional signal processing. This is based on defining the blockwise partial moments, some moment relations for the cepstrum etc. The usual calculation of moments of a discrete signal requires a summation of weighted signal samples. However, it is also possible to compute these moments using the frequency domain expressions. In the present work, we will show new moment relations based on its frequency domain interpretation which have strong applications in homomorphic signal processing. They allow ideal calculation even if it is done for infinite duration signals. The usual signal analysis methods in homomorphic processing involve Fourier analysis for infinite duration signal sequences. This leads to unavoidable errors like aliasing and the essentil use of sophisticated algorithms like phase unwrapping etc. It is shown that the calculation of moments for complex cepstrum of signal sequences can be made even without calculating the actual cepstral coefficients. Then, these moments are used to describe the cepstrum. This

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implicit but ideal way of describing the cepstrum has merit when

1) a direct analysis of infinite duration cepstrum is required. A description of cepstrum is made in terms of its moments which are calculated ideally from the original sequence.

2) a direct computation of cepstrum is required only as an intermediate step. In these cases, ideal calculation of moments of complex cepstral sequence in terms of moments of original sequence is done, thereby obviating the need for calculation of cepstral sequence necessarily.

Applications of this method have been considered in problems of homomorphic deconvolution. In the usual linear filtering also, an analysis of linear phase sequences is presented which are interpreted as product of Fourier transforms of corresponding minimum phase and all pass sequences.

II. MOMENTS OF DISCRETE TIME SIGNAL

In signal processing, moments of the continuous and discrete time signals are used in several analyses [7], [9]. For the discrete signals, the usual calculation of the moment of a signal sequence generally involves computation of the sum of the weighted signal sequence. The weights depend upon order and the type of the moment. For example, the simple geometric moments use the sample distance from a arbitrary origin raised to a certain power as the weight for that signal sample. These time domain definitions exist for both one dimensional and two dimensional signals. In this paper, we shall be primarily concerned with the signals in one dimension and their simple geometric moments. We recall the geometric moment's time domain definition and show its extension in frequency domain.

2.1 Time Domain Definition

The standard definition for the r-th order geometric moment m_r in time for a discrete time signal x (iT) becomes follows.

$$m_r = \sum_{i=-\infty}^{\infty} (iT)^r x(iT) \qquad (1)$$

Here, T is sampling peiod and x (iT) is the i-th sample of the signal. The origin in the above definition is assumed to be zero. By changing the origin, we can define several other momets. However, changing the origin of the moment changes its value. Moments about different origins are related to each other as follows [10].

$$m_r(t_2) = \sum_{i=0}^{r} {}_rC_i (t_1 - t_2)^{r-i} m_r(t_1) \qquad (2)$$

Here rCi is the total number of different combinations of i objects from r objects. t_1 and t_2 are the two arbitrary origins.

If the signal is not known explicitly in time domain, moment evaluation using Eq.(1) is not possible.

There are two other cases when an evaluation using Eq.(1) is not possible. one is when the moment of a certain order does not exist. In this case, moments of higher orders also do not exist [10] and we cannot do much about it. The other and more interesting case occurs when the signal in Eq.(1) extends upto infinity. It is quite clear from Eq.(1) that if the signal duration is infinite as is the case in IIR filters, a direct evaluation is again not possible. This becomes the case even if the moment of different orders are known to exist. In the next section, we show that using the moment definition in the frequency domain, calculation of moment is possible if the Fourier transform of the signal is known in a convenient closed form or is known to be related to the Fourier transform of some other signal whose moments can be calculated easily.

2.2 Frequency Domain Definition

If we use the frequency domain expressions, the calculation of moment is done by evaluating the derivatives of the Fourier transform at the zero frequency.

If X $(e^{j\omega T})$ represents the Fourier transform of the discrete signal sequence x (iT),

$$m_r = j^r \frac{\partial^r X(e^{j\omega T})}{\partial \omega^r}\bigg|_{\omega=0} \tag{3}$$

In other words, Fourier transform is the moment generating function by evaluating the derivatives of the Fourier transform at zero frequency. This has the same meaning of a characteristic function defined in statistics [10]. Note that we have used the partial derivative of the Fourier transform to emphasize the fact that we are interested in its derivatives with respect to ω alone. Note also that for real signals, moment calculation using Eq.(3) also yields real values. Starting from this frequency domain evaluation of the Fourier transform derivatives for calculation of the moments, we shall now show certain very interesting applications of this in homomorphic signal processing, cepstral analysis and deconovolution problems. We start from the cepstrum of a signal sequence. Our approach will be based on calculating the moments of signal sequence whose frequency spectrum is known in closed form or is related to some other signal sequence, moments of which can be calculated easily. We shall relate the moments of those two sequences. From now onwards we shall assume $T=1$ to simplify the expressions.

III. COMPLEX CEPSTRUM

3.1 Moments of cepstrum

Given a signal sequence x (n), it is of considerable interest to calculate its complex cepstrum and to examine its properties [11], [12]. The calculation of the cepstrum, however involves the use of sophisticated algorithms like phase unwrapping [13] etc. Using the DFT or FFT algorithms also, it is not possible to ascertain the cepstrum of a signal without having the effects of aliasing error. This is due to the fact that even if the original signal is of finite duration, its cepstrum will be of infinite duration in general. For the differential cepstrum [17], the phase unwrapping may be avoided but the problem of aliasing remains. It is shown that the moments for a signal sequence and its corresponding cepstral sequence are related in a way which obviates the need for direct calculation of cepstral coefficients. Hence an ideal calculation for the moment of the cepstrum is possible even if the duration of the cepstrum is infinite [8]. From now onward, we shall call the moment of the cepstrum as the "Cepstral Moment".

Let us start from the basic definition of the cepstrum. If X $(e^{j\omega})$ represents the Fourier transform of the original signal x (n), same of the complex cepstrum, $\hat{X}$ $(e^{j\omega})$ is given by

$$\hat{X}(e^{j\omega}) = \log\left[X(e^{j\omega})\right] \tag{4}$$

Interpretation of this logarithmic function requires care and well discussed in the literature [14]. Assuming that this has been done properly, we now employ our frequency domain expression to calculate the moments of the cepstral sequence. The key to this method lies in fact that differentiating the logarithm of a function generates the expressions which involves the derivatives of the orginal function in the numerator and in the denominator. If we denote $\hat{m}_r$ as the cepstral moment of order r and m_r as the moment of the original sequence, we obtain the following relations.

$$\hat{m}_0 = \log[m_0] \tag{5}$$

$$\hat{m}_1 = \frac{m_1}{m_2} \tag{6}$$

$$\hat{m}_2 = \frac{m_2}{m_0} - \frac{(m_1)^2}{(m_0)^2} \tag{7}$$

$$\hat{m}_3 = \frac{m_3}{m_0} - \frac{3m_2m_1}{(m_0)^2} + \left[\frac{2(m_1)^3}{(m_0)^3}\right] \tag{8}$$

[We will not need the actual value of $\hat{m}_0$, so only $m_0 \neq 0$ is assumed.]
Expressions for the higher order moments can also be derived similarly.

It means that if we can calculate the moments of the original signal sequence, we can determine the cepstral moments using the above expressions. Consider, for example, the case when the original signal sequence is real, stable and is of finite duration. It is easy to see that in this case all of its moments of finite order exist and can be calculated using the time domain expression, Eq.(1). Since the cepstrum of such sequence is real and in general, of infinite duration, ascertaining the cepstral moments without calculating the explicitly is of great value.

3.2 Example calculation of cepstral moments

Applying the above method, we now illustrate the actual calculation of cepstral moments of certain useful sequences.
As the first example, let us consider the following minimum phase sequence.

$$\begin{aligned} x(n) &= a^n \quad n \geqq 0 \\ &= 0 \quad n < 0 \end{aligned} \qquad (9)$$

where $|a| < 1$.

The cepstrum of this sequence is given by the following [15].

$$\begin{aligned} \hat{x}(n) &= a^n/n \qquad n > 0 \\ &= 0 \qquad n \leqq 0 \end{aligned} \qquad (10)$$

If we calculate $\hat{m}_0$ and $\hat{m}_1$ etc. using the direct expression for $\hat{x}(n)$, we obtain the following.

$$\begin{aligned} m_0 &= \sum_{n=1}^{\infty} a^n/n = -\log(1-a) \\ &= \log[1/(1-a)] \end{aligned} \qquad (11)$$

$$\begin{aligned} m_1 &= \sum_{n=1}^{\infty} n \cdot a^n/n \\ &= a/(1-a) \end{aligned} \qquad (12)$$

Next, we calculate these moments from the moments of the original sequence x (n).
For this sequence, m_0, m_1 ect. become as follows.

$$\begin{aligned} m_0 &= \sum_{n=0}^{\infty} a^n \\ &= 1/(1-a) \end{aligned} \qquad (13)$$

$$\begin{aligned} m_1 &= \sum_{n=0}^{\infty} n \cdot a^n \\ &= a/(1-a)^2 \end{aligned} \qquad (14)$$

Then, the cepstral moments become as

$$\begin{aligned} \hat{m}_0 &= \log[m_0] = \log[1/(1-a)] \\ &= -\log[1-a] \end{aligned} \qquad (15)$$

and

$$\begin{aligned} \hat{m}_1 &= m_1/m_0 \\ &= [a/(1-a)^2] \cdot (1-a) \\ &= a/(1-a) \end{aligned} \qquad (16)$$

The two methods yield the identical result showing the fact that in order to calculate the cepstral moments of a sequence, we need to know only the moments of the original sequence. It is not necessary to calculate the cepstrum explicitly.

As a second example, let us consider the following nonminimum phase sequence.

$$\begin{aligned} x(n) &= b^n \qquad 0 \leqq n \leqq M-1 \\ &= 0 \qquad \text{otherwise} \end{aligned} \qquad (17)$$

where $|b| > 1$.

The cepstrum of this sequence is given by [15]

$$\begin{aligned} \hat{x}(n) = \log[b^{M-1}]\delta(n) &+ \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} (1-M)[(\cos\pi k)/k]\delta(n-k) \\ &+ \sum_{k=-\infty}^{-1} (b^{Mk}/k)\delta(n-kM) \\ &- \sum_{k=-\infty}^{-1} (b^k/k)\delta(n-k) \end{aligned} \qquad (18)$$

Moments of the original sequence are readily calculated as

$$m_0=(b^M-1)/(b-1) \tag{19}$$

$$m_1=\frac{[M(b-1)b^M-b(b^M-1)]}{(b-1)^2} \tag{20}$$

Other moments can also be derived similarly.
Using the expressions developed for the cepstral moments, we get

$$\begin{aligned}\hat{m}_0&=\log[m_0]\\&=\log(b^{M-1})-\log(b-1)\\&=\log(b^{M-1})+\log(1-b^{-M})\\&\quad-\log(1-b^{-1})\end{aligned} \tag{21}$$

$$\begin{aligned}\hat{m}_1&=m_1/m_0\\&=M/(1-b^{-M})-1/(1-b^{-1})\end{aligned} \tag{22}$$

These moments are verified using the direct expression for the cepstrum given by Eq.(18).

IV. CUMULANTS OF DISCRETE TIME SIGNALS

In statistics, we have the concept of cumulant [10] which has the interesting property of remaining invariant under the change of origin point. The definition of the cumulant of order r, k_r for the signal x (n) is given by the following series expansion of the Fourier transform X $(e^{j\omega})$.

$$X(e^{j\omega})=\exp(\sum_{r=0}^{\infty} k_r(-j\omega)^r/r!) \tag{23}$$

While the defining equation is presented in the above form, it is more useful to consider $\log[X(e^{j\omega})]$. Then Eq. (23) becomes

$$\log[X(e^{j\omega})]=\sum_{r=0}^{\infty} k_r(-j\omega)^r/r!) \tag{24}$$

If we consider log $[X(e^{j\omega})]$ as the Fourier transform of the corresponding and use the Maclaurin's series for $e^{-j\omega}$

$$\begin{aligned}\hat{X}(e^{j\omega})&=\log[X(e^{j\omega})]\\&=\sum_{r=0}^{\infty}\hat{m}_r(-j\omega)^r/r!)\end{aligned} \tag{25}$$

Here $\hat{m}_r$ is the cepstral moment of order r. A comparison of this equation with Eq. (24) reveals the interesting result that the cepstral moments are the same as cumulants of the original sequence.

$$\hat{m}_r=k_r \qquad \text{for all r.} \tag{26}$$

However cumulants, unlike moments, are not ascertainable directly through common summatory or integrative processes. One method involves the use of Fourier transform in closed form and its series expansion. However, as is obvious, this method is of very limited practical interest. Another method of practical value is to find moments first and then employ linear relations to find the cumulants. This is well discussed in [10] but we will just mention one recursive relation from reference.

$$\frac{\partial m_i}{\partial k_1}=i\cdot m_{i-1} \qquad i>0 \tag{27}$$

The above relation has the implicit assumption that the moment of order zero is normalized to 1. In general, this is not the case, therefore the proper multiplication factor must be used.

$$m_0=\exp(k_0) \tag{28}$$

Using the above relations, it is possible to generate a whole set of moment expressions in terms of the cumulants [10]. Therefore, if the cumulants of a signal sequence are known, we can compute the corresponding moments.

V. LINEAR PHASE AND MINIMUM PHASE SEQUENCE

In this section, we show some moment relations for the minimum phase sequences derived from their corresponding linear phase sequences.

let any real linear phase sequence be denoted by $x_{lin}(n)$ and its Fourier transform be $X_{lin}(e^{j\omega})$. It is well known that $X_{lin}(e^{j\omega})$ can be expressed as the product of Fourier transform of the corresponding minimum phase sequence and the all pass sequence. Thus

$$X_{lin}(e^{j\omega}) = X_{min}(e^{j\omega}) \cdot X_{ap}(e^{j\omega}) \qquad (29)$$

where $X_{min}(e^{j\omega})$ and $X_{ap}(e^{j\omega})$ are the Fourier transform of the minimum phase and all pass sequences respectively.

Taking logarithm of both the sides with the proper justification for log function assumed

$$\log\,[X_{lin}(e^{j\omega})] = \log\,[X_{min}(e^{j\omega})] + \log[X_{ap}(e^{j\omega})] \qquad (30)$$

which implies further as

$$\hat{X}_{lin}(e^{j\omega}) = \hat{X}_{min}(e^{j\omega}) + \hat{X}_{ap}(e^{j\omega}) \qquad (31)$$

where $\hat{}$ represents the Fourier transform of the cepstrum.

Then, using the linearity of moments with respect to the Fourier transform

$$\hat{m}_r(\mathrm{lin}) = \hat{m}_r(\mathrm{ap}) + \hat{m}_r(\mathrm{min}) \qquad (32)$$

where $\hat{m}_r$ ( ) represents the moment of the corresponding cepstral sequence.

Using the explicit expression for the frequency spectrum of the ideal all pass systems

$$X_{ap}(e^{j\omega}) = e^{j\phi(\omega)} \qquad (33)$$

Taking logarithm of both the sides, we have

$$\log[X_{ap}(e^{j\omega})] = j\phi(\omega) \qquad (34)$$

Thus, the cepstral moments of the all pass system are given by

$$\hat{m}_r(\mathrm{ap}) = j^{r+1} \left. \frac{\partial^r \phi(\omega)}{\partial \omega^r} \right|_{\omega=0} \qquad (35)$$

However, the cepstrum of real all pass sequence must be real. In fact, cepstrum of all real sequences must be real. Therefore cepstral moments of the all pass sequence must also be real. Since j^{r+1} is imaginary for the even values of r, it can be concluded that the even order cepstral moments of the ideal all pass system must be zero. A useful property of the real all pass systems can be stated as following.

"Even order cepstral moments of real all pass sequences are always zero."

Using this result in Eq. (32), we get the following.

$$\hat{m}_r(\mathrm{lin}) = \hat{m}_r(\mathrm{min}) \qquad \text{for r even.} \qquad (36)$$

It is also possible to get at this result intuitively by realizing the fact that the magnitudes of the Fourier transform of the minimum phase and linear phase sequence are identical in Eq.(29).

While the above shows just a theoretical relation between minimum phase and linear phase sequence, the approach of calculating the moments of the cepstrum first and then treating these moments as the cumulants of the original sequence does have interesting practical applications. In reference to the design of minimum phase FIR filters [16], it will be discussed in a future communication.

VI. HOMOMORPHIC DECONVOLUTION

6.1 Moment of convolved sequence

In homomorphic deconvolution problems [11], [15] we are basically concerned with the process of separating the components of a convolution output using the homomorphic processing. However, this method requires computation of cepstral sequence to realize the homomorphic system. If it is possible to compute this sequence in a closed form, then the deconvolution can carried out without any aliasing or approximation error. But as is often the case, a closed-form expression for the resulting cepstrum is not possible in many practical problems. Using the results of the previous sections, we describe a new method for deconvolution which is based on homomorphic processing but avoids the explicit calculation of the cepstrum and the use of sophisticated algorithms like phase unwrapping etc. It also leads to ideal calculation, free from aliasing error etc. which results due to inverse FFT computation for a infinite duration sequence. This ideal calculation is possible not only for the minimum phase sequences but also for the general sequences having their moments calculable ideally.

Our method is based on the following steps.

(1) Compute the moments of output sequence and the sequence to be removed.

(2) Compute the moments for the cepstrum of these two sequences using the expressions described in setion 3.1. Note that this calculation is ideal even if the length of cepstral sequence is infinite.

(3) Compute the moments of the cepstral sequence of the sequence to be recovered. This is simply the difference of the two moments calculated in step (2).

(4) Realize the fact that these moments are in fact the cumulants for the sequence to be recovered.

(5) Compute moments for the sequence to be recovered from its cumulants.

(6) Using the inverse relatioms for the sequence values in terms of the moments, compute the sequence to be recovered. Table 1 shows some of these inverse matrices for different lengths of the sequence. Further details of this are to be found in ref.[18].

For the convolution of the two sequence x_1 (n) and x_2 (n), the output sequence y (n) is given by

$$y(n)=x_1(n) * x_2(n) \tag{37}$$

let us suppose all the sequences are real finite duration sequences. Let us suppose also that y (n) and $x_2(n)$ are known. This is a strong assumption and further work is going on to estimate the moments of $x_2(n)$ by some other means.

Then using the steps described above, we arrive at the following expressios. Here, $m_r(x_1)$denotes moment of order r of sequence x_1 etc.

$$m_0(x_1)=m_0(y)/m_0(x_2) \tag{38}$$

$$m_1(x_1)=[m_1(y)/m_0(x_2)] -m_0(y)m_1(x_2)/[m_0(x_2)]^2 \tag{39}$$

$$m_2(x_1)=\frac{m_2(y)}{m_0(x_2)}-\frac{m_0(y)m_2(x_2)}{[m_0(x_2)]^2}+\frac{2m_0(y)[m_1(x_2)]^2}{[m_0(x_2)]^3}-\frac{2m_1(y)m_1(x_2)}{[m_0(x_2)]^2} \tag{40}$$

Similarly, the other expressions follow.

Expressions for the higher order moments can also be written or directly programmed for actual computation. Once we know the required number of moments, we can compute the sequence $x_1(n)$ using the inverse matrix for the required length. Note that the above equations are valid for the convolution of the two sequences in general and do not depend upon the properties of the sequences.

Thus, whether for finite duration or infinite duration, these moment relations always exist.

6.2 Computer simulation results

We have simulated the above procedure on a computer by creating a long sequence divided into the segments so that maximum order of moment required for each segment is 15. This restriction can be relaxed to permit higher lengths of the segment if the accuracy of the given computer is higher.

A long sequence x_1 (n) of length 512 was created randomly. It was convolved with a short sequence $x_2(n)$ of length 3 to produce an output sequece y (n). Table 2 shows the actual signal samples of this segment of x_1 (n), both by the direct calculation and using the results described above. In order to be

Table 1 Inverse Moment Coefficient Matrix for some values of N

N=1 $\quad [1] \quad$ N=2 $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

N=3 $\begin{bmatrix} 1 & -3/2 & 1/2 \\ 0 & 2 & -1 \\ 0 & -1/2 & 1/2 \end{bmatrix}$

N=4 $\begin{bmatrix} 1 & -11/6 & 1 & -1/6 \\ 0 & 3 & -5/2 & 1/2 \\ 0 & -3/2 & 2 & -1/2 \\ 0 & 1/3 & -1/2 & 1/6 \end{bmatrix}$

N=5 $\begin{bmatrix} 1 & -25/12 & 35/24 & -5/12 & 1/24 \\ 0 & 4 & -13/3 & 3/2 & -1/6 \\ 0 & -3 & 19/4 & -2 & 1/4 \\ 0 & 4/3 & -7/3 & 7/6 & -1/6 \\ 0 & -1/4 & 11/24 & -1/4 & 1/24 \end{bmatrix}$

Table 2 Signal sample values after deconvolution

sample number	calculated sample value	actul sample value
15	−9.962307242375E+000	−9.962304687500E+000
14	−4.044311612282E+000	−4,043554687500E+000
13	2.545739779320E−001	2.476953125000E−001
12	4.255183126813E+000	4.281445312500E+000
11	7.366883754826E+000	7.321445312500E+000
10	−6.618569010054E+000	−6.703554687500E+000
9	5.197452333675E+000	5.020820312500E+000
8	6.885615853367E+000	6.582070312500E+000
7	8.558466156539E+000	8.079570312500E+000
6	−3.093642164209E+000	−3.095429687500E+000
5	−7.714322826663E−001	−6.416796875000E−001
4	1.687971707935E+000	1.711445312500E+000
3	6.324999715337E+000	6.293320312500E+000
2	−6.307361530577E+000	−6.310429687500E+000
1	1.272529223796E+000	1.272695312500E+000
0	−1.024348393855E+001	−1.024355468750E+001

able to appreciate the error estimates, we defined the following criterion.

$$\varepsilon=(\Delta x)^2=\frac{\{\sum_n |x_{ca}(n)-x_{ac}(n)|^2\}}{\sum_n |x_{ac}(n)|^2} \tag{41}$$

The values obtained for ε are of the order of 0,5 percent.

6.3 Echo removal

To be able to appreciate the potential of the moment method, we consider the process of echo removal. For this, let us consider a general impulse train described by the following equation.

$$p(n)=\delta(n)+\sum_{K=1}^{M-1}\alpha_k\delta(n-n_k) \tag{42}$$

Sequences of this kind can be used to represent signals that contain echoes or reverberations. For example, the sequence whose values are

$$y(n)=x(n)+\sum_{K=1}^{M-1}\alpha_k x(n-n_k) \tag{43}$$

can be represented as the convolution

$$y(n)=x(n)*p(n) \tag{44}$$

In the conventional method, one has to assume that echoes are spaced uniformly and all the results then are for this special case. The uniformly spaced impulse train is given by

$$p(n)=\sum_{K=0}^{M-1}\alpha_k\delta(n-kn_0) \tag{45}$$

The uniformly-spaced impulse train is equivalent to the case of a signal echo. In this special case, the complex cepstrum turns out to be an impulse train at the same spacing. Fig.1(a) and 1(b) illustrate the two cases. If the spacing is not uniform, the conventional method fails, because it is not possible to relate the impulses in the resulting complex cepstrum with that of the spacing of the echoes [15]. If the impulse train is nonminimum phase, then also the computation of complex cepstrum becomes extremely complicated. The merit in using the conventional method is that it requires only the partial infomation about the longest duration of echo. It then uses an appropriate frequency-invariant linear system to dereverberate the sequence. However, the problems discussed in section associated with the calculation of the complex cepstrum remain, not allowing the ideal error-free deconvolved signal using the conventional method. However, it is very easy to calculate the moments of this kind of an impulse train, and hence moments of impulse train.

First few moments of this impulse train become as follows.

$$m_0(p)=1+\sum_{K=0}^{M-1}\alpha_k \tag{46}$$

$$m_1(p)=\sum_{K=0}^{M-1}\alpha_k n_k \tag{44}$$

$$m_2(p)=\sum_{K=0}^{M-1}\alpha_k[n_k]^2 \tag{48}$$

The degree of complexity in applying the moment method remains essentially the same, irrespective of the nature of the spacing or the minimum/non-minimum phase sequences. Thus the moment method is superior to the usual homomorphic processing method in this aspect in addition to the merit that it does not involve any calculation of the cepstrum. But, it requires more information about the echo to be removed and is a demerit. Howerver, we are trying to investigate the procedure to estimate the longest duration of the echo and moments of the echo of a given channel by some other means and will be discussed later.

6.4 Computational aspect

In ref. [2], a very fast method to generate the first 16 moments of a two dimensional signal is described using an all-pole filter. In actual problems, we shall not require moments of order more than 16. So, we shall use the above method for calculating the moments. Though the higher order systems do pose a computational problem regarding moments, systems with moderate orderate order upto 50 are easily programmed for the computer realization. With the higher order systems, the main difficulty occurs in handling the large values of the moments. However, the significance of an ideal solution obtainable in terms of moments, has nevertheless its value. One technique to overcome such problems is to divide the long sequences into several smaller subsequences which allows the calculation accurately on a given computer. We can also consider, for example, the techniques like Overlap Add Method and Overlap Save Methods etc.

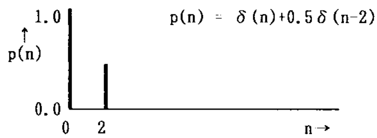

Fig. 1(a) Single echo

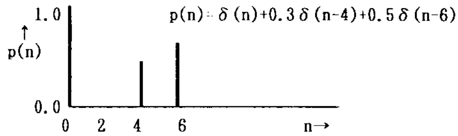

Fig. 1(b) Nonuniformly-spaced echo train

VII. CONCLUSIONS

The geometric moments of a signal sequence are known to be related to the derivatives of the Fourier transform of the signal sequence. It so turns out that in many signal processing problems, we require computation of the cepstrum of the sequence as sn intermediate step towards the final solution. Since the duration of the cepstrum is in general infinite even for a real causal sequence of finite duration, use of normal processing methods like FFT etc. leads to aliasing errors. It also requires sophisticated algorithms like phase unwrapping etc. To deal with the problems of this kind, a new method based on moments has been presented. Though the calculation of the moments of very high order is restricted due to large computational error on computers, the strong merit of this method is the fact that moments of the infinite duration cepstrum of a real, causal and finite duration sequence can be computed ideally, without actually finding the cepstrum explicitly. It is being tried to break the long sequences into smaller ones and then be able to apply this method. As an intermediate step towards the final solution, use is made of the inverse relations which describe the sequence in terms of its moments. To illustrate the application of this, some cepstral moment relations for the linear phase and its corresponding minimum phase sequence have been established. Several examples in the problems of deconvolution and echo removal using the homomorphic processing have also been considered.

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